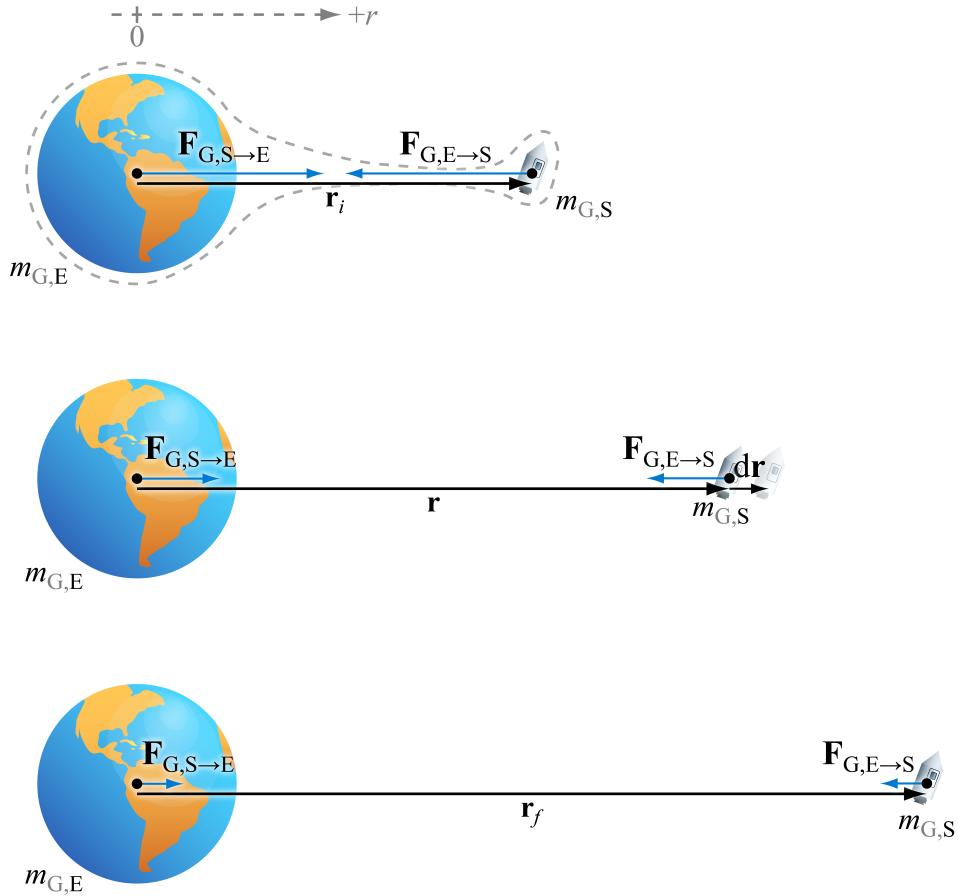


$\frac{1}{r^2}$ fields have $\frac{1}{r}$ potentials

Gravitational potential energy



$$-\Delta U_{G,E\&S} = \underbrace{\Delta W_{G,S \rightarrow E}}_0 + \Delta W_{G,E \rightarrow S}$$

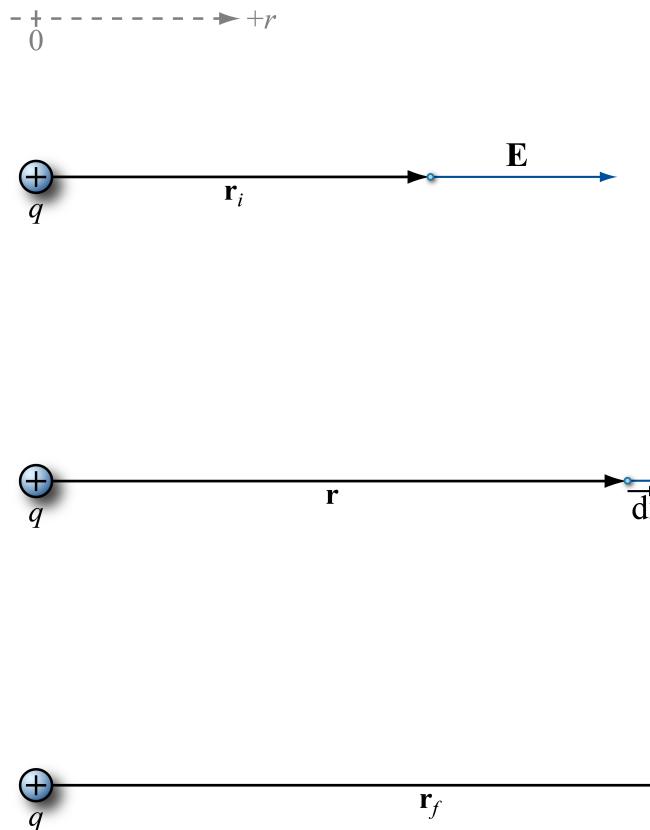
$$\begin{aligned}\Delta U_{G,E\&S} &= -\Delta W_{G,E \rightarrow S} \\ &= - \int_{\substack{\vec{r}=\vec{r}_f \\ \vec{r}=\vec{r}_i \\ r=r_f \\ r=r_i}} \vec{F}_{G,E \rightarrow S} \cdot d\vec{r} \\ &= - \int_{r=r_i}^{r=r_f} \frac{Gm_E m_S}{r^2} (-\hat{\mathbf{r}}) \cdot dr \hat{\mathbf{r}} \\ &= Gm_E m_S \int_{r=r_i}^{r=r_f} \frac{1}{r^2} \hat{\mathbf{r}} \cdot \hat{\mathbf{r}} dr \\ &= Gm_E m_S \int_{r=r_i}^{r=r_f} r^{-2} dr \\ &= Gm_E m_S \left[\frac{r^{-1}}{-1} \right]_{r=r_i}^{r=r_f} \\ \Delta U_{G,E\&S} &= - \underbrace{\frac{Gm_E m_S}{r_f}}_{U_{G,E\&S,f}} - \left(- \underbrace{\frac{Gm_E m_S}{r_i}}_{U_{G,E\&S,i}} \right)\end{aligned}$$

$$U_{G,E\&S} = -\frac{Gm_E m_S}{r}$$

$\frac{1}{r^2}$ fields have $\frac{1}{r}$ potentials

Electrostatic potential

$$\Delta V = \int_{\vec{r}=\vec{r}_i}^{\vec{r}=\vec{r}_f} -\vec{E} \cdot d\vec{r}$$



$$\begin{aligned}\Delta V &= \int_{\vec{r}=\vec{r}_i}^{\vec{r}=\vec{r}_f} -\frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \cdot dr \hat{r} \\ &= -\frac{q}{4\pi\epsilon_0} \int_{\vec{r}=\vec{r}_i}^{\vec{r}=\vec{r}_f} \frac{1}{r^2} \underbrace{\hat{r} \cdot \hat{r}}_1 dr \\ &= -\frac{q}{4\pi\epsilon_0} \int_{\vec{r}=\vec{r}_i}^{\vec{r}=\vec{r}_f} r^{-2} dr \\ &= -\frac{q}{4\pi\epsilon_0} \left[\frac{r^{-1}}{-1} \right]_{r=r_i}^{r=r_f} \\ \Delta V &= \frac{1}{4\pi\epsilon_0} \frac{q}{r_r} - \frac{1}{4\pi\epsilon_0} \frac{q}{r_i} \\ &\quad \text{"V(\vec{r}_f)"} \quad \text{"V(\vec{r}_i)"}\end{aligned}$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$